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Vortex pair creation on brane-antibrane pair via marginal deformation

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Abstract: It has been conjectured that the vortex solution on a D-brane anti-D-brane system represents a D-brane of two lower dimension. We establish this result by first identifying a series of marginal deformations which create the vortex-antivortex pair on the brane-antibrane system, and then showing that under this series of marginal deformations the original D-brane anti-D-brane system becomes a D-brane anti-D-brane system with two lower dimensions. Generalization of this construction to the case of solitons of higher codimension is also discussed.

Keywords: D-branes, Conformal Field Models in String Theory.

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Contents

1. Introduction and summary

BPS D-brane anti-D-brane system of type-IIA and IIB string theories admit tachyonic modes [1, 2]. It has been conjectured that at the minimum of the tachyon potential the tension of the brane-antibrane system is exactly canceled by the negative value of the tachyon potential, so that at this point the brane-antibrane system is indistinguishable from the vacuum [3, 4]. It has been further conjectured that various solitonic solutions on the brane-antibrane pair, where the tachyon approaches the minimum of the potential asymptotically, represent various lower-dimensional branes. Thus for example, on a single Dp-$\bar{\text{D}}p$ brane system, the kink solution represents a non-BPS D$(p-1)$-brane [5]–[9] of type-II string theory, whereas a vortex solution represents a BPS D$(p-2)$-brane of the same theory [6, 10]. There are generalizations of this conjecture in which solitons of higher codimension on more than one pair of D-brane $\bar{\text{D}}$-brane system correspond to BPS and non-BPS D-branes of codimension > 2 [6, 10, 9].[1]

The codimension one case, i.e. the identification of the kink solution on the Dp-brane $\bar{\text{D}}p$-brane pair with a non-BPS D$(p-1)$-brane, has been demonstrated explicitly (although indirectly) by identifying a series of marginal deformations which

[1]Recently evidence for some of these conjectures, and similar conjectures [11, 12] involving D-branes in bosonic string theories, have been found [13]–[18] using string field theory [19, 20]. This approach uses the level truncation scheme developed by Kostelecky and Samuel [21, 22]. These conjectures have also been analysed using renormalization group flow on the world-sheet theory [23, 24] following earlier work of ref. [25].

interpolate between the D-brane $\bar{\text{D}}$-brane pair and the kink solution, and showing that this series of deformations take the original Dp-brane $\bar{\text{D}}p$-brane system to a non-BPS D$(p-1)$-brane [6, 26]. This is done by compactifying one of the directions tangential to the brane-antibrane system on a circle, switching on half unit of Wilson line on the antibrane, and reducing the radius of the circle to a critical radius where the lowest mode of the tachyon becomes marginal. One then uses this marginal deformation to create the kink and then takes the radius of the circle back to infinity.

It is natural to ask if this procedure can be generalised to the case of vortex and higher codimension solitons on the brane-antibrane pair. One faces the following problem for the vortex. The original Dp-brane $\bar{\text{D}}p$-brane system is neutral under Ramond-Ramond (RR) gauge fields since the RR charge of the brane and the antibrane cancel. But a vortex, being identified to a BPS D$(p-2)$-brane, carries RR charge. Since RR charge is quantized, one cannot hope to have a continuous interpolation between these two configurations.[2] What one can hope to do however is to find a marginal deformation which interpolates between the original Dp-brane $\bar{\text{D}}p$-brane system and a vortex-antivortex pair on this system. If we can show that this marginal deformation converts the boundary conformal field theory (BCFT) associated with the Dp-brane $\bar{\text{D}}p$-brane system to the BCFT associated with the D$(p-2)$-brane $\bar{\text{D}}(p-2)$-brane system, then we would establish the equivalence between a vortex solution and a D$(p-2)$-brane.

This is the problem we address in this paper. The steps involved in the analysis, which have been summarised in section 2, are more or less the same as the ones used for showing the equivalence of the kink solution on the brane-antibrane pair with a codimension one non-BPS brane. For convenience of notation we study the case of a vortex solution on a D2-brane $\bar{\text{D}}$2-brane system. We compactify both directions tangential to the brane, switch on appropriate Wilson lines, and reduce the radii of the torus to certain critical values where the tachyonic deformation corresponding to the creation of the vortex-antivortex pair becomes a marginal deformation. We then switch on this marginal deformation and study the fate of the BCFT under this marginal deformation. These steps have been discussed in detail in section 3. In section 4 we study the effect of increasing the radii of the compact directions back to large values. We show that under this series of deformations the original BCFT gets deformed to a new BCFT describing the dynamics of open strings on a D0-$\bar{\text{D}}$0 pair. This establishes the equivalence of a vortex solution on a D2-brane $\bar{\text{D}}$2-brane pair, and a D0-brane. In section 5 we discuss generalization of this analysis to solutions of higher codimension.

[2]It has been suggested by Maldacena that by switching on background magnetic field as in ref. [27] one may be able to resolve this problem [28]. The approach followed in [29, 30] might also be helpful.

2. The general strategy

In this section we shall outline the general strategy that we shall follow in order to establish the equivalence between a vortex-antivortex pair on a BPS Dp-brane $\bar{\mathrm{D}}p$-brane system, and a D$(p-2)$ $\bar{\mathrm{D}}(p-2)$-brane pair. For definiteness we shall take our starting point to be a 2-brane $\bar{2}$-brane pair of type-IIA string theory, but the analysis can clearly be generalised to any p.

The steps are as follows:

1. We take a parallel 2-brane $\bar{2}$-brane pair of type-IIA string theory along $x^1 - x^2$ plane, and compactify the x^1 and x^2 directions on circles of radii R_1 and R_2, respectively. There are four different Chan-Paton (CP) sectors of open string states. States with CP factors proportional to the 2×2 identity matrix I and the Pauli matrix σ_3, representing open string states with both ends on the same brane, have the standard GSO projection under which the Neveu-Schwarz (NS) sector ground state is taken to be odd. On the other hand states with CP factors σ_1 and σ_2, representing open strings with two ends on two different branes, have opposite GSO projection, and in particular contain tachyonic modes. We shall identify the tachyonic modes associated with the sectors σ_1 and σ_2 with the real and the imaginary parts respectively of a complex tachyon field T.

2. We switch on half a unit of Wilson line along each of these circles. This makes open strings with CP factors σ_1 and σ_2, including the tachyon field T, antiperiodic along each of the two circles. Thus we can expand the tachyon field as
$$T\left(x^1, x^2, t\right) = \sum_{m,\, n \in \mathbb{Z}} T_{m+\frac{1}{2}, n+\frac{1}{2}}(t)\, e^{i(m+\frac{1}{2})\frac{x^1}{R_1} + i(n+\frac{1}{2})\frac{x^2}{R_2}}\,. \tag{2.1}$$
The mass of the mode $T_{m+\frac{1}{2}, n+\frac{1}{2}}$ is given by
$$\left(M_{m+\frac{1}{2}, n+\frac{1}{2}}\right)^2 = \frac{\left(m+\frac{1}{2}\right)^2}{(R_1)^2} + \frac{\left(n+\frac{1}{2}\right)^2}{(R_2)^2} - \frac{1}{2}\,, \tag{2.2}$$
in the $\alpha' = 1$ unit that we shall be using.

3. From eq. (2.2) we see that for $(R_1)^{-2} + (R_2)^{-2} > 2$ there are no tachyonic modes. For $(R_1)^{-2} + (R_2)^{-2} = 2$ the modes $T_{\pm\frac{1}{2}, \pm\frac{1}{2}}$ become marginal. We shall see in section 3 that at $R_1 = R_2 = 1$ the deformation corresponding to
$$\begin{aligned} T\left(x^1, x^2\right) &= -i\,\alpha\left[e^{\frac{i}{2}(x^1+x^2)} - e^{-\frac{i}{2}(x^1+x^2)} + i\left(e^{\frac{i}{2}(x^1-x^2)} - e^{-\frac{i}{2}(x^1-x^2)}\right)\right] \\ &= 2\,\alpha\left(\sin\frac{x^1+x^2}{2} + i\sin\frac{x^1-x^2}{2}\right), \end{aligned} \tag{2.3}$$

for arbitrary constant α becomes exactly marginal, i.e. switching on this vev of the tachyon does not cost any energy. $T(x^1, x^2)$ has zeroes at $x^1 = x^2 = 0$ and at $x^1 = x^2 = \pi$. Near $x^1 = x^2 = 0$,

$$T \simeq (1+i)\,\alpha\left(x^1 - ix^2\right). \tag{2.4}$$

Thus it looks like a vortex.[3] On the other hand, near $x^1 = x^2 = \pi$,

$$T \simeq (-1+i)\,\alpha\left(\left(x^1 - \pi\right) + i\left(x^2 - \pi\right)\right). \tag{2.5}$$

This looks like an anti-vortex. Thus we see that switching on a tachyon background of the form (2.3) corresponds to creating a vortex-antivortex pair.

4. After switching on the deformation (2.3) we take the radii R_1 and R_2 back to infinity, since we want to describe a vortex-antivortex pair on infinite D2-brane $\bar{\text{D}}$2-brane system. Here we find that for a generic value of α, once we increase R_1, R_2 beyond 1, there is a one point function of the modes $T_{\pm\frac{1}{2},\pm\frac{1}{2}}$. This is not surprising, since for $R_i > 1$, the deformation (2.3) is a relevant perturbation, and hence switching on this background breaks conformal invariance of the world-sheet theory. However, we find that besides the trivial background $\alpha = 0$, there is another inequivalent point (which we shall choose to be $\alpha = 1$ by suitably normalizing α) where the one point function of $T_{\pm\frac{1}{2},\pm\frac{1}{2}}$ vanish, and hence the theory is conformally invariant. We shall identify the $\alpha = 1$ point as the vortex-antivortex pair.

   As we shall show in section 4, increasing the x^1 and x^2 radii to arbitrary values R_1 and R_2 in the presence of $\alpha = 1$ background can be described equivalently as increasing the radii of a pair of different coordinates $\widehat{x}^1$ and $\widehat{x}^2$. But in these coordinates, the original D2-$\bar{\text{D}}$2 brane system, after being deformed by the tachyon background (2.3) with $\alpha = 1$, appears as a D0-$\bar{\text{D}}$0 brane system situated at diametrically opposite points of the torus spanned by $\widehat{x}^1$ and $\widehat{x}^2$. This shows that the spectrum of open strings in the background of a vortex-antivortex pair on the D2-$\bar{\text{D}}$2 system wrapped on a torus with radii (R_1, R_2) is identical to that on a D0-$\bar{\text{D}}$0 brane system situated on a torus with the same radii. This establishes the equivalence between the vortex-antivortex pair on a D2-$\bar{\text{D}}$2 brane system and D0-$\bar{\text{D}}$0 brane pair.

3. Conformal field theory at the critical radii

In this section we shall study the marginal deformation of the BCFT on the upper half plane at the critical radii $R_1 = R_2 = 1$ by (2.3). The relevant part of the BCFT at the critical radii before we switch on the perturbation (2.3) is described by a pair

[3] Of course whether we call this a vortex or an anti-vortex is a matter of convention. Once we fix the convention for the vortex, then a configuration with opposite orientation can be identified as an anti-vortex.

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of scalar fields $X^i \equiv X^i_L + X^i_R$ for $i = 1, 2$ and their right- and left-moving fermionic superpartners χ^i_R, χ^i_L. The Neumann boundary condition satisfied by these fields on the real line is given by

$$X^i_L = X^i_R \equiv \frac{1}{2}\, X^i_B\,, \qquad \chi^i_L = \chi^i_R \equiv \chi^i_B\,. \tag{3.1}$$

We are considering here the NS sector open string states. In the Ramond (R) sector we have a different boundary condition on χ^i, but it can be handled in a manner similar to the one discussed in ref. [6] and will not be discussed here.

Besides these fields we also have a time coordinate and its superpartners satisfying Neumann boundary condition and 7 other space-like coordinates and their superpartners satisfying Dirichlet boundary condition. We shall refer to these fields as spectator fields as they do not play a major role in the dynamics of the problem. We also have fermionic ghost fields b_L, c_L, b_R, c_R and bosonic ghost fields $\beta_L, \gamma_L, \beta_R, \gamma_R$ satisfying Neumann boundary condition. We shall denote by Φ_L, Φ_R the left- and right-moving bosonized ghost field of the β, γ system [31], satisfying the boundary condition

$$\Phi_L = \Phi_R \equiv \Phi_B\,. \tag{3.2}$$

The tachyon vertex operator in the (-1) picture [31] corresponding to the deformation (2.3) is given by:

$$\begin{aligned} V_T^{(-1)} \propto e^{-\Phi_B} \Big[&\Big(e^{\frac{i}{2}(X^1_B + X^2_B)} - e^{-\frac{i}{2}(X^1_B + X^2_B)} \Big) \otimes \sigma_1 + \\ &+ \Big(e^{\frac{i}{2}(X^1_B - X^2_B)} - e^{-\frac{i}{2}(X^1_B - X^2_B)} \Big) \otimes \sigma_2 \Big]\,. \end{aligned} \tag{3.3}$$

In "zero" picture this vertex operator takes the form:

$$\begin{aligned} V_T^{(0)} \propto \Big[&\left(\chi^1_B + \chi^2_B\right) \Big(e^{\frac{i}{2}(X^1_B + X^2_B)} + e^{-\frac{i}{2}(X^1_B + X^2_B)} \Big) \otimes \sigma_1 + \\ &+ \left(\chi^1_B - \chi^2_B\right) \Big(e^{\frac{i}{2}(X^1_B - X^2_B)} + e^{-\frac{i}{2}(X^1_B - X^2_B)} \Big) \otimes \sigma_2 \Big]\,. \end{aligned} \tag{3.4}$$

Let us now define a new set of variables:

$$\begin{aligned} Y^1 &= \frac{1}{\sqrt{2}} \left(X^1 + X^2\right), \qquad & Y^2 &= \frac{1}{\sqrt{2}} \left(X^1 - X^2\right), \\ \psi^1_R &= \frac{1}{\sqrt{2}} \left(\chi^1_R + \chi^2_R\right), \qquad & \psi^2_R &= \frac{1}{\sqrt{2}} \left(\chi^1_R - \chi^2_R\right), \\ \psi^1_L &= \frac{1}{\sqrt{2}} \left(\chi^1_L + \chi^2_L\right), \qquad & \psi^2_L &= \frac{1}{\sqrt{2}} \left(\chi^1_L - \chi^2_L\right). \end{aligned} \tag{3.5}$$

These fields satisfy the boundary conditions:

$$Y^i_L = Y^i_R \equiv \frac{1}{2}\, Y^i_B\,, \qquad \psi^i_L = \psi^i_R \equiv \psi^i_B\,, \quad (i = 1, 2) \tag{3.6}$$

on the real line. In terms of these fields,

$$V_T^{(0)} \propto \left[\psi_B^1 \left(e^{\frac{i}{\sqrt{2}}Y_B^1} + e^{-\frac{i}{\sqrt{2}}Y_B^1}\right) \otimes \sigma_1 + \psi_B^2 \left(e^{\frac{i}{\sqrt{2}}Y_B^2} + e^{-\frac{i}{\sqrt{2}}Y_B^2}\right) \otimes \sigma_2\right]. \tag{3.7}$$

We now fermionize the scalar fields Y^i as follows:

$$e^{i\sqrt{2}\,Y_R^i} = \frac{1}{\sqrt{2}}\left(\xi_R^i + i\,\eta_R^i\right) \otimes \tau_i\,, \qquad e^{i\sqrt{2}\,Y_L^i} = \frac{1}{\sqrt{2}}\left(\xi_L^i + i\,\eta_L^i\right) \otimes \tau_i\,, \tag{3.8}$$

where ξ_R^i, η_R^i (ξ_L^i, η_L^i) are right- (left-) moving Majorana-Weyl fermions, and the Pauli matrices τ_i denote cocycle factors [32] which must be put in to guarantee correct (anti-)commutation relations between various fields.[4] We also attach a cocycle factor τ_3 to $\psi_{L,R}^i$ and all the spectator fermions. This guarantees for example that ψ^i and the spectator fermions commute with both sides of eq. (3.8). The cocycle factors should be taken to commute with CP factors.

We can find another representation of the same conformal field theory by re-bosonising the fermions as follows:

$$\frac{1}{\sqrt{2}}\left(\xi_R^i + i\,\psi_R^i\right) = e^{i\sqrt{2}\,\phi_R^i} \otimes \widetilde{\tau}_i\,, \qquad \frac{1}{\sqrt{2}}\left(\xi_L^i + i\psi_L^i\right) = e^{i\sqrt{2}\,\phi_L^i} \otimes \widetilde{\tau}_i\,. \tag{3.9}$$

ϕ^1, ϕ^2 represent a pair of free scalar fields, and $\widetilde{\tau}_i$ are a new set of cocycle factors. In this convention $\eta_{L,R}^1$, $\eta_{L,R}^2$ and spectator fermions carry the new cocycle factor $\widetilde{\tau}_3$. There is a third representation in which we use a slightly different rebosonization:

$$\frac{1}{\sqrt{2}}\left(\eta_R^i + i\,\psi_R^i\right) = e^{i\sqrt{2}\,\phi_R^{i\prime}} \otimes \widehat{\tau}_i\,, \qquad \frac{1}{\sqrt{2}}\left(\eta_L^i + i\,\psi_L^i\right) = e^{i\sqrt{2}\,\phi_L^{i\prime}} \otimes \widehat{\tau}_i\,, \tag{3.10}$$

where $\phi^{i\prime}$ is another pair of scalar fields, and $\widehat{\tau}_i$ denote another set of cocycle factors. $\xi_{L,R}^1$, $\xi_{L,R}^2$ and the spectator fermions will carry the cocycle factor $\widehat{\tau}_3$ when we use the set of variables $\phi^{i\prime}$, ξ^i and the spectator fields to describe the BCFT. We also define

$$\phi^i = \phi_L^i + \phi_R^i\,, \qquad \phi^{i\prime} = \phi_L^{i\prime} + \phi_R^{i\prime}\,. \tag{3.11}$$

For later use we list here the operator product expansions, and the relations between the currents of free fermions and bosons:

$$\psi_R^i(z)\,\psi_R^j(w) \simeq \xi_R^i(z)\,\xi_R^j(w) \simeq \eta_R^i(z)\,\eta_R^j(w) \simeq \frac{i}{z-w}\,\delta_{ij}\,, \tag{3.12}$$

$$\partial Y_R^i(z)\,\partial Y_R^j(w) \simeq \partial\phi_R^i(z)\,\partial\phi_R^j(w) \simeq \partial\phi_R^{i\prime}(z)\,\partial\phi_R^{j\prime}(w) \simeq -\frac{1}{2(z-w)^2}\,\delta_{ij}\,, \tag{3.13}$$

$$\psi_R^i\,\xi_R^i = i\sqrt{2}\,\partial\phi_R^i\,, \qquad \eta_R^i\,\xi_R^i = i\sqrt{2}\,\partial Y_R^i\,, \qquad \psi_R^i\,\eta_R^i = i\sqrt{2}\,\partial\phi_R^{i\prime}\,, \tag{3.14}$$

[4]As we shall be using these bosonization formulae to manipulate vertex operators in the BCFT on the upper half plane, we only need to require that correct (anti-)commutation relations are satisfied by the vertex operators subject to the boundary condition (3.6).

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with no summation over i in the last equation. Here $\simeq$ denotes equality up to non-singular terms. There are also similar relations involving the left-moving currents.

Using eq. (3.8) the boundary condition (3.6) on Y^i can be translated to the following boundary condition on the fermions:

$$\xi^i_L = \xi^i_R \equiv \xi^i_B\,, \qquad \eta^i_L = \eta^i_R \equiv \eta^i_B\,. \tag{3.15}$$

Combining these with the boundary condition (3.6) on ψ^i, we see from (3.9) and (3.10) that ϕ^i and $\phi^{i\prime}$ both satisfy Neumann boundary condition on the real line

$$\phi^i_L = \phi^i_R \equiv \frac{\phi^i_B}{2}\,, \qquad \phi^{i\prime}_L = \phi^{i\prime}_R \equiv \frac{\phi^{i\prime}_B}{2}\,. \tag{3.16}$$

Using the bosonization relations (3.8)–(3.10) and (3.14), the vertex operator $V_T^{(0)}$ given in eq. (3.7) can be expressed as

$$V_T^{(0)} \propto \Big[\psi^1_B\,\xi^1_B\otimes\sigma_1\otimes\tau_2 - \psi^2_B\,\xi^2_B\otimes\sigma_2\otimes\tau_1\Big] \propto \Big[\partial\phi^1_B\otimes\sigma_1\otimes\tau_2 - \partial\phi^2_B\otimes\sigma_2\otimes\tau_1\Big]\,. \tag{3.17}$$

∂ denotes tangential derivative along the boundary. The two operators appearing in the right-hand side of eq. (3.17) correspond to vertex operators of constant U(1) gauge field along ϕ^1 and ϕ^2 directions, with generators $\sigma_1\times\tau_2$ and $\sigma_2\times\tau_1$, respectively. Since these two matrices commute, we see that switching on the tachyon vev corresponds to switching on a pair of commuting U(1) Wilson lines A_{ϕ_1} and A_{ϕ_2} along the bosonic directions ϕ^1 and ϕ^2. This represents an exactly marginal deformation. Switching on finite vacuum expectation value (vev) of the tachyon then corresponds to inserting the operator

$$\exp\left(\frac{i\,\alpha}{2\sqrt{2}}\oint\partial\phi^1_B\otimes\sigma_1\otimes\tau_2 - \frac{i\,\alpha}{2\sqrt{2}}\oint\partial\phi^2_B\otimes\sigma_2\otimes\tau_1\right), \tag{3.18}$$

where $\oint$ denotes integration along the boundary. Note that we have fixed the normalization of α in a specific manner. This is the same normalization convention as in ref. [6], and so we shall be able to use the results of ref. [6] directly. With this normalization, α is a periodic variable with period 2.

Before we study the effect of switching on such a tachyon vev on the spectrum of open strings, let us study the open string spectrum before switching on the tachyon vev. Let us denote by $\mathcal{H}$ the Hilbert space of states[5] created by the half-integer moded ψ^i, ξ^i, η^i oscillators and the spectator fields on the SL$(2,\mathbb{R})$ invariant vacuum $|0\rangle$. Alternatively we can also view $\mathcal{H}$ as the Hilbert space of states created from $|0\rangle$ by the spectator fields, together with vertex operators involving ψ^i and Y^i with Y^i momenta quantized in units of $1/\sqrt{2}$, or vertex operators involving η^i and ϕ^i with ϕ^i

[5]Here we are discussing open string states; so all states are created from $|0\rangle$ by vertex operators inserted at the boundary of the upper half plane.

CP $\otimes$ cocycle	$(-1)^F$	h_1	h_2	A_{ϕ^1}	A_{ϕ^2}
$I \otimes I$	$+$	$+$	$+$	n	n
$I \otimes \tau_3$	$+$	$-$	$-$	c	c
$\sigma_3 \otimes I$	$+$	$+$	$+$	c	c
$\sigma_3 \otimes \tau_3$	$+$	$-$	$-$	n	n
$\sigma_1 \otimes \tau_1$	$-$	$+$	$-$	c	c
$\sigma_1 \otimes \tau_2$	$-$	$-$	$+$	n	n
$\sigma_2 \otimes \tau_1$	$-$	$+$	$-$	n	n
$\sigma_2 \otimes \tau_2$	$-$	$-$	$+$	c	c

Table 1: Quantum numbers carried by open string states with various CP and cocycle factors. The last two columns indicate whether the open string states are neutral (n) or charged (c) under the gauge fields A_{ϕ^1} and A_{ϕ^2}. Note that not all combinations of CP$\otimes$ cocycle factors are present in the spectrum.

momenta quantized in units of $1/\sqrt{2}$, or vertex operators involving ξ^i and $\phi^{i\prime}$ with $\phi^{i\prime}$ momenta quantized in units of $1/\sqrt{2}$. On $\mathcal{H}$, we denote by $(-1)^F$ the world-sheet fermion number which changes the sign of ψ^1, ψ^2, and the spectator fermions, leaving ξ^i, η^i (and hence Y^i) and the spectator bosons unchanged. The $\mathrm{SL}(2,\mathbb{R})$ invariant ground state $|0\rangle$ is taken to be odd under $(-1)^F$. We also define the transformations h_1 and h_2 as follows. Both h_1 and h_2 leave ψ^1, ψ^2 and all the spectator fermions unchanged, but h_1 changes the sign of ξ^1, η^1 and h_2 changes the sign of ξ^2, η^2. In terms of the bosonic variables X^i or Y^i,

$$\begin{aligned}
h_1 : X^1_{L,R} &\longrightarrow X^1_{L,R} + \frac{\pi}{2}\,, \qquad X^2_{L,R} \longrightarrow X^2_{L,R} + \frac{\pi}{2}\,, \\
Y^1_{L,R} &\longrightarrow Y^1_{L,R} + \frac{\pi}{\sqrt{2}}\,, \qquad Y^2_{L,R} \longrightarrow Y^2_{L,R}\,, \\
h_2 : X^1_{L,R} &\longrightarrow X^1_{L,R} + \frac{\pi}{2}\,, \qquad X^2_{L,R} \longrightarrow X^2_{L,R} - \frac{\pi}{2}\,, \\
Y^1_{L,R} &\longrightarrow Y^1_{L,R}\,, \qquad Y^2_{L,R} \longrightarrow Y^2_{L,R} + \frac{\pi}{\sqrt{2}}\,.
\end{aligned} \tag{3.19}$$

It is easy to verify that with this definition, $(h_1)^2$ and $(h_2)^2$ act as identity on all states. States with CP factor I and σ_3 are $h_1 h_2$ even and $(-1)^F$ even, whereas states with CP factors σ_1 and σ_2 are $h_1 h_2$ odd and $(-1)^F$ odd. Taking into account the assignment of the cocycle factors (τ_1 for h_1 odd states, τ_2 for h_2 odd states and τ_3 for $(-1)^F$ odd states) the quantum numbers carried by various open string states, when expressed in terms of ψ^i, η^i, ξ^i and the spectator fields, are as given in table 1.[6]

[6]There will be extra cocycle factors involving $\widetilde{\tau}_i$ when we express the vertex operators in terms

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The complete spectrum of open string states carrying a given CP$\otimes$cocycle factor is obtained by keeping all states in $\mathcal{H}$ carrying the quantum numbers mentioned in the table. The last two columns describe if the open string state is charged or neutral under the gauge fields A_{ϕ^1} or A_{ϕ^2}. This is easily determined by examining if the cocycles factors commute with $\sigma_1 \otimes \tau_2$ and $\sigma_2 \otimes \tau_1$, since these represent the U(1) generators associated with the background gauge fields.

From table 1 we note that before we switch on the background (3.18), each state is invariant under $(-1)^F h_1 h_2$, and that all combinations of the quantum numbers subject to this condition appear exactly twice in this table. Thus the combined spectrum from all the sectors contain two copies of $\mathcal{H}$ with $(-1)^F h_1 h_2$ projection. We also note that open string states carrying CP$\otimes$cocycle factors $I \otimes I$, $\sigma_3 \otimes \tau_3$, $\sigma_1 \otimes \tau_2$ and $\sigma_2 \otimes \tau_1$ are all neutral under both gauge fields. Combining the spectrum from all four neutral sectors we see that each combination of quantum numbers (subject to the condition $(-1)^F h_1 h_2 = +1$) appears exactly once; thus the combined spectrum of charge neutral states contain a single copy of $\mathcal{H}$ with $(-1)^F h_1 h_2$ projection. The combined spectrum of the charged states also contains a single copy of $\mathcal{H}$ with $(-1)^F h_1 h_2$ projection.

Let us now define a new set of symmetry generators $(-1)^{F_\phi}$, h_{ϕ^1} and h_{ϕ^2} on $\mathcal{H}$ by using the representation where $\mathcal{H}$ is generated by action on $|0\rangle$ by the vertex operator involving ϕ^i, η^i and the spectator fields. $(-1)^{F_\phi}$ changes the signs of η^i and the spectator fermions, leaving unchanged ϕ^i and the spectator bosons, and has eigenvalue -1 acting on the SL$(2,\mathbb{R})$ invariant vacuum $|0\rangle$. h_{ϕ^i} leaves unchanged η^1, η^2 and all the spectator fields, and transform ϕ^j as follows:

$$\begin{aligned} h_{\phi^1} &: \phi^1_{L,R} \longrightarrow \phi^1_{L,R} + \frac{\pi}{\sqrt{2}}\,, \qquad \phi^2_{L,R} \longrightarrow \phi^2_{L,R}\,, \\ h_{\phi^2} &: \phi^2_{L,R} \longrightarrow \phi^2_{L,R} + \frac{\pi}{\sqrt{2}}\,, \qquad \phi^1_{L,R} \longrightarrow \phi^1_{L,R}\,. \end{aligned} \tag{3.20}$$

Thus h_{ϕ^i} changes the sign of ξ^i and ψ^i. With this definition, and using the bosonization relations (3.8) and (3.9), it is easy to see that

$$(-1)^F\, h_1\, h_2 = (-1)^{F_\phi}\, h_{\phi^1}\, h_{\phi^2}\,. \tag{3.21}$$

(Both sides change the sign of $\xi^{1,2}$, $\eta^{1,2}$ and $\psi^{1,2}$, and the spectator fermions, leaving the spectator bosons unchanged.) Thus the combined spectrum of the charge neutral states as well as the combined spectrum of the charged states may be identified as a copy of $\mathcal{H}$, subject to the $(-1)^{F_\phi} h_{\phi^1} h_{\phi^2}$ projection.

Upon switching on the gauge fields, the spectrum in the charge neutral sector does not change, but the spectrum in the charged sector changes since the quantization laws for the ϕ^i momenta p_{ϕ^i} change. By combining the fields into charge

of the fields ϕ^i, η^i and the spectator fields. But since these cocycle factors commute with those carried by the gauge fields A_{ϕ^1} and A_{ϕ^2}, we can ignore them for the purpose of studying how the spectrum changes when we switch on the gauge fields.

eigenstates we can follow the change in p_{ϕ_1}, p_{ϕ_2} as a function of α and thus completely determine the spectrum at every value of α.[7] However as we shall see in the next section, once we deform the radius away from $R_1 = R_2 = 1$ point, $\alpha = 0$ and 1 are the only inequivalent points which give conformally invariant theories. Since $\alpha = 0$ represents the trivial tachyon background, we shall be interested in the $\alpha = 1$ point. The spectrum at $\alpha = 1$ simplifies enormously if we notice that this corresponds to shifting the p_{ϕ^1} and p_{ϕ^2} quantization laws by $\pm 1/\sqrt{2}$ [6], so that its effect is to simply reverse the sign of the h_{ϕ^1} and h_{ϕ^2} quantum numbers. But the initial spectrum did not contain h_{ϕ^1} and h_{ϕ^2} projections individually. It only contained $(-1)^{F_\phi} h_{\phi^1} h_{\phi^2}$ projection, and this does not change. As a result the spectrum at $\alpha = 1$ is identical to the spectrum at $\alpha = 0$!

It may appear from this that at the end of the deformation the system has come back to the original system! However, as we shall see in section 4, the response of this system to a change in the radii R_1 and R_2 is very different from that of the original system. In order to facilitate the analysis of section 4, we introduce dual coordinates

$$\widehat{X}^i_L = X^i_L\,, \qquad \widehat{X}^i_R = -X^i_R\,. \tag{3.22}$$

In terms of the new coordinates $\widehat{X}^i$, the BCFT at $\alpha = 1$ corresponds to a D0-brane $\bar{\text{D}}$0-brane pair, situated at diametrically opposite points of a square torus with unit radii.

4. Deforming away from the critical radius

In this section we shall consider the effect of switching on the perturbation that deforms the radii away from their critical values. The procedure followed here will be similar to the one used in ref. [6], so our discussion will be brief. There are four possible marginal deformations of the bulk conformal field theory, three of which correspond to deformation of the shape and size of the torus labelled by the X^1-X^2 coordinates, and the fourth one corresponds to switching on the anti-symmetric tensor field in the X^1-X^2 plane. Using eqs. (3.5) we can express a general perturbation of this kind in the (0,0) picture as

$$K_{ij}\, \partial X^i_L\, \partial X^j_R = a_{ij}\, \partial Y^i_L\, \partial Y^j_R\,, \tag{4.1}$$

where

$$a = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} K \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}. \tag{4.2}$$

[7] With the normalization of α chosen here, $\alpha = 2$ corresponds to a shift in p_{ϕ^i} by $\pm\sqrt{2}$. This corresponds to states carrying same h_{ϕ^1} and h_{ϕ^2} quantum numbers. Hence α and $\alpha + 2$ describes the same BCFT.

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First we shall consider the effect of first order perturbation, and show that in the presence of this perturbation the tachyon vertex operator V_T develops a one point function unless $\alpha = 0$ or $\alpha = 1$. The procedure for doing this is similar to that discussed in ref. [6]. We insert a tachyon vertex operator $V_T^{(0)}$ given in eq. (3.17) at a point on the boundary of the disk (or upper half plane), the background (3.18) at the boundary of the disk, and a closed string vertex operator corresponding to the perturbation (4.1) in the $(-1,-1)$ picture at the center of the disk. This is proportional to

$$a_{ij}\, e^{-\Phi_L}\, e^{-\Phi_R}\, \psi_L^i\, \psi_R^j\,, \tag{4.3}$$

where Φ_L and Φ_R are the left- and right-moving bosonized ghost fields [31]. Computation of this amplitude is straightforward using the description of the BCFT in terms of the (η^i, ϕ^i) fields [6]. The ϕ^i momentum conservation laws tell us that only the $i = j$ terms in (4.3) contribute. The answer is proportional to

$$(a_{11} + a_{22}) \sin(\pi\alpha)\,. \tag{4.4}$$

This shows that this one point function vanishes only for $\alpha = 0$ and $1(\mathrm{mod}2)$. It is also easy to check that at these two values of α the one point function of all other open string vertex operators also vanish, and hence these configurations describe consistent boundary conformal field theories.

Next we need to study what happens when we go beyond the lowest order perturbation in the deformation parameters a_{ij}. For this let us consider an amplitude with an arbitrary number of insertions of the operator (4.1) at various points in the interior of the disk, insertion of the background (3.18) with $\alpha = 1$ at the boundary of the disk, and a set of open string vertex operators, whose correlation function we want to calculate, at the boundary of the disk. We take two of these open string vertex operators to be in the -1 picture and the rest in the zero picture so that the total picture number of all the vertex operators add up to -2. Again the calculation proceeds as in ref. [6]. We express all the operators in terms of η^i, ϕ^i and the spectator fields. The vertex operator (4.1) can be expressed as products of η_L^i, η_R^i, $e^{\pm i\sqrt{2}\phi_L^i}$ and $e^{\pm i\sqrt{2}\phi_R^i}$ fields. The integrated vertex operator (3.18) inserted at the boundary has two effects. It effectively shifts the ϕ^i momenta of the open string vertex operators as discussed in section 3, and $\oint \partial\phi_B^i$ picks up the ϕ^i winding number of all the closed string vertex operators inserted in the interior of the disk. Using the operator product expansion (3.13) it is easy to verify that $\frac{1}{2\sqrt{2}} \oint \partial\phi_B^i = \frac{1}{\sqrt{2}} \oint \partial\phi_R^i$, acting on any combination of closed string vertex operators, gives an integral multiple of π. Now, since

$$\exp\left(i\pi n\, \sigma_1 \otimes \tau_2\right) = \exp(i\pi n)\,, \qquad \exp\left(i\pi n\, \sigma_2 \otimes \tau_1\right) = \exp(i\pi n)\,, \quad \text{for integer } n \tag{4.5}$$

we see that in taking into account the effect of (3.18) for $\alpha = 1$ on the closed string vertex operators we can replace it by

$$\exp\left(\frac{i}{2\sqrt{2}}\oint \partial\phi_B^1 - \frac{i}{2\sqrt{2}}\oint \partial\phi_B^2\right) = \exp\left(\frac{i}{\sqrt{2}}\oint \partial\phi_R^1 - \frac{i}{\sqrt{2}}\oint \partial\phi_R^2\right). \tag{4.6}$$

In going from the left-hand side to the right-hand side of eq. (4.6) we have used the boundary condition (3.16). Since ϕ_R^i are holomorphic fields, we can now deform the integration contour into the interior of the disk, picking up residues from the location of the closed string vertex operators. The net effect is to replace each insertion of (4.1) by

$$\exp\left(\frac{i}{\sqrt{2}}\oint \partial\phi_R^1 - \frac{i}{\sqrt{2}}\oint \partial\phi_R^2\right) a_{ij}\,\partial Y_L^i\,\partial Y_R^j\,, \tag{4.7}$$

where the contours of integration in the exponent are around the location of the closed string vertex operator. This can be easily evaluated using eqs. (3.9)–(3.14), and the result is[8]

$$-a_{ij}\,\partial Y_L^i\,\partial Y_R^j = -K_{ij}\,\partial X_L^i\,\partial X_R^j\,. \tag{4.8}$$

In terms of the coordinates $\widehat{X}^i$ introduced in eq. (3.22), the right-hand side of eq. (4.8) can be expressed as

$$K_{ij}\,\partial\widehat{X}_L^i\,\partial\widehat{X}_R^j\,. \tag{4.9}$$

This has the same form as the left-hand side of (4.1) with X^i replaced by $\widehat{X}^i$. Thus deforming the X^i radius from 1 to R_i in the presence of the tachyon background (3.18) with $\alpha = 1$ corresponds to deforming the $\widehat{X}^i$ radius from 1 to R_i. The D-brane system now describes a D0-brane $\bar{\text{D}}$0-brane pair situated at diametrically opposite points of this torus.

This shows that the spectrum of open string states living on a vortex antivortex pair at the diametrically opposite points on a D2-$\bar{\text{D}}$2 system wrapped on a torus is identical to the spectrum of open string states living on a D0-$\bar{\text{D}}$0 brane pair at the diametrically opposite points of the same torus. This leads to the identification of the vortex (antivortex) on a D2-$\bar{\text{D}}$2 system with a D0 ($\bar{\text{D}}$0) brane.

5. Generalizations

The method used here can easily be generalized to prove the identification of a vortex solution on a Dp-$\bar{\text{D}}p$ brane pair with a D$(p-2)$-brane. The analysis is exactly

[8] Here we have differed somewhat from the analysis of ref. [6]. In [6] $\oint\partial\phi_B$ in the exponent was replaced by $\oint(\partial\phi_L + \partial\phi_R)$ instead of $2\oint\partial\phi_R$. If we follow this procedure here, then eq. (4.8) will be replaced by $-a_{ij}\,\partial\phi_L^{i\prime}\,\partial\phi_R^{j\prime}$. This would be analogous to the corresponding result in [6]. The final result for the spectrum and correlation functions however does not depend on which procedure we follow, since the two procedures are related by the symmetry transformation $(\phi_L^i, \phi_R^i) \to (\phi_L^i + \frac{\pi}{2\sqrt{2}}, \phi_R^i + \frac{\pi}{2\sqrt{2}})$.

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identical; all that is required is to change the Dirichlet boundary condition on $(p-2)$ of the spectator superfields to Neumann boundary condition.

It is also possible to generalize this analysis to show that a codimention $2n$ soliton on 2^{n-1} pairs of Dp-$\bar{\mathrm{D}}p$ brane system represents a D$(p-2n)$-brane [6, 10]. Again for simplicity let us assume that all the spectator fields except the time direction has Dirichlet boundary condition, i.e. take $p = 2n$. We compactify each of the $2n$ directions tangential to the D-branes on a circle of radius 1. Let us label these coordinates by $x^1, \ldots, x^{2n}$. The open string states carry $2^n \times 2^n$ CP factors; with the tachyon states carrying off diagonal CP factors of the form:

$$\begin{pmatrix} 0 & A \\ A^\dagger & 0 \end{pmatrix}, \tag{5.1}$$

where A is a $2^{n-1} \times 2^{n-1}$ complex matrix. Under the $U(2^{n-1}) \times U(2^{n-1})$ gauge transformation on the brane-antibrane system, generated by $U(2^{n-1})$ matrices U and V,

$$A \longrightarrow U\, A\, V^\dagger\,. \tag{5.2}$$

Let us now, following ref. [10], choose a $(2^n \times 2^n)$-dimensional representation of the $2n$-dimensional Clifford algebra in which each of the Γ-matrices has the form given in (5.1). Let X^i $(1 \le i \le 2n)$ denote the coordinate fields tangential to the D-brane, and χ^i_R, χ^i_L be their right- and left-moving superpartners. Let us define

$$\begin{aligned} Y^{2k+1} &= \frac{1}{\sqrt{2}}\left(X^{2k+1} + X^{2k+2}\right), & Y^{2k+2} &= \frac{1}{\sqrt{2}}\left(X^{2k+1} - X^{2k+2}\right), \\ \psi^{2k+1}_R &= \frac{1}{\sqrt{2}}\left(\chi^{2k+1}_R + \chi^{2k+2}_R\right), & \psi^{2k+2}_R &= \frac{1}{\sqrt{2}}\left(\chi^{2k+1}_R - \chi^{2k+2}_R\right), \\ \psi^{2k+1}_L &= \frac{1}{\sqrt{2}}\left(\chi^{2k+1}_L + \chi^{2k+2}_L\right), & \psi^{2k+2}_L &= \frac{1}{\sqrt{2}}\left(\chi^{2k+1}_L - \chi^{2k+2}_L\right), \end{aligned} \tag{5.3}$$

for $0 \le k \le (n-1)$. Consider now the following vertex operator in the (-1) picture:

$$V_T^{(-1)} \propto e^{-\Phi_B} \sum_{i=1}^{2n} \sin\frac{Y^i_B}{\sqrt{2}} \otimes \Gamma_i\,, \tag{5.4}$$

where the subscript B denotes the boundary value as usual. In order that this is an allowed vertex operator, we need to switch on appropriate Wilson lines on the branes; we assume that this has been done. (We can, for example, take the Wilson line along the X^{2k+1} and the X^{2k+2} direction to be $\Gamma^{2k+1}\Gamma^{2k+2}$. Conjugation by this matrix changes the sign of Γ^{2k+1} and Γ^{2k+2}, keeping the other Γ^i's invariant. Unlike in the case analysed earlier, this amounts to switching on Wilson lines both on the D-branes and the anti-D-branes.) In the zero picture the vertex operator (5.4) takes the form:

$$V_T^{(0)} \propto \sum_{i=1}^{2n} \psi^i_B \cos\frac{Y^i_B}{\sqrt{2}} \otimes \Gamma_i\,. \tag{5.5}$$

It is easy to see via the bosonization procedure that each term in the sum represents switching on a constant gauge field (Wilson line) along a new bosonic coordinate and hence is a marginal deformation. Furthermore, since all the Γ matrices anti-commute with each other, and the fermions ψ^i_B also anti-commute with each other, we see that the different terms in the sum commute with each other. After bosonization this will imply that the CP$\otimes$cocycle factors carried by the different gauge fields commute with each other. Thus (5.5) corresponds to switching on constant, commuting gauge fields along different directions, and represents a marginal deformation.

In analogy with the analysis of sections 3, 4, one expects that the BCFT at the end of the marginal deformation (the $\alpha = 1$ point) is more naturally described in terms of a set of dual coordinates $\widehat{x}^i$ in which the system appears as 2^{n-1} D0-$\bar{\text{D}}$0 brane pair. If we increase the radii of the original torus to arbitrary values R_i after the marginal deformation, then it would correspond to increasing the radii of the torus described by the $\widehat{x}^i$ coordinates to the same values R_i as in section 4. Thus the end result will be 2^{n-1} D0-$\bar{\text{D}}$0 brane pairs situated at different point on the torus with radii R_i.[9]

On the other hand, we can examine the tachyon background (5.4) and try to identify it as a collection of solitons. For this we need to identify the soliton cores as the places where the tachyon field vanishes. This requires each y^i to be an integral multiple of $\pi\sqrt{2}$. Using eqs. (5.3), and that each x^i describes a circle of radius 1, we see that the inequivalent points are given by all possible combination of the configurations

$$\left(x^{2k+1}, x^{2k+2}\right) = (0,0) \qquad \text{or} \qquad (\pi,\pi)\,. \tag{5.6}$$

Since there are n pairs of coordinates, this gives 2^n possibilities. To study the nature of the solution near the core, let us consider one of them, e.g. $(x^1, \ldots, x^{2n}) = (0, \ldots, 0)$. The tachyon background near this point is proportional to

$$y^i\,\Gamma_i\,. \tag{5.7}$$

This is precisely the form suggested in ref. [10] for a codimension $2n$ soliton on 2^{n-1} brane-antibrane pairs. On the other hand, expanding the tachyon field near the point $(x^1, x^2, \ldots, x^{2n}) = (\pi, \pi, 0, 0, \ldots, 0)$, we get the tachyon background to be proportional to:

$$-\left(y^1 - \pi\sqrt{2}\right)\Gamma^1 + \sum_{i=2}^{2n} y^i\,\Gamma^i\,. \tag{5.8}$$

[9] A consistency check for this scenario is that at the critical radii the total mass of the initial configuration is given by $2^n/g$, where g denotes the closed string coupling constant, since each of the 2^n D-$2n$-branes / $\bar{\text{D}}$-$2n$-branes has area equal to $(2\pi)^{2n}$ and tension equal to $1/((2\pi)^{2n}g)$. This agrees with the total mass of 2^{n-1} D0-$\bar{\text{D}}$0 brane pair. Also both the initial and the final state has vanishing RR charge.

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The $-$ sign in front of the first term indicates that it has opposite orientation compared to (5.7) and represents an antisoliton. In general the solution around a point (5.6) represents a soliton (antisoliton) if the number of coordinate pairs taking the value (π, π) is even (odd). Thus the tachyon background (5.5) represents creation of 2^{n-1} soliton-antisoliton pairs. From this we can conclude that 2^{n-1} soliton-antisoliton pairs represent 2^{n-1} D0-$\bar{\rm D}$0 pairs, and hence a single D0-brane should be identified with the single codimension $2n$ soliton on the D-$2n$-brane $\bar{\rm D}$-$2n$-brane pair.

Generalization of this result to solitons of odd codimension, which are expected to describe non-BPS D-branes [10, 9] is also straightforward. In this case we take 2^{n-1} D-$(2n-1)$-brane $\bar{\rm D}$-$(2n-1)$-brane pairs in type-IIB string theory along $x^1, \ldots, x^{2n-1}$, with $x^1, \ldots, x^{2n-2}$ directions compactified on circles of radii 1 and x^{2n-1} direction compactified on a circle of radius $1/\sqrt{2}$. We define Y^i's for $1 \leq i \leq (2n-2)$ as in eq. (5.3) and consider tachyon background associated with the vertex operator[10]

$$V_T^{(-1)} \propto e^{-\Phi_B} \left[\sum_{i=1}^{2n-2} \sin \frac{Y_B^i}{\sqrt{2}} \otimes \Gamma_i + \sin \frac{X_B}{\sqrt{2}} \otimes \Gamma_{2n-1} \right] . \tag{5.9}$$

This corresponds to creation of 2^{n-1} solitons at

$$\begin{aligned} \left(x^{2k+1}, x^{2k+2}\right) = (0,0) \qquad \text{or} \qquad (\pi, \pi) \quad \text{for } 0 \leq k \leq (n-2) \, , \\ x^{2n-1} = 0 \, . \end{aligned} \tag{5.10}$$

On the other hand it is easy to see that the vertex operator (5.9), in the zero picture, represents a marginal deformation. We expect this to convert the original brane configuration to a configuration of non-BPS branes as in ref. [6]. The number of such D0-branes can easily be seen to be 2^{n-1} by comparing the masses of the initial and the final configurations. This shows the equivalence between the non-BPS D0-brane and a soliton on 2^{n-1} D-$(2n-1)$ $\bar{\rm D}$-$(2n-1)$ brane pairs.

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